Modified Distribution Method (MODI method) or U-V method

- for checking optimal solution.

Numerical: find an optimal solution

Factory \ Warehouses →	W_1	W_2	W_3	W_4	Availability ↓
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	08	70	20	18
Requirement →	5	8	7	14	(34)

Solution: First solve above transportation problem with Vogel's Approximation Method and find the initial basic feasible solution. (IBFS)

As we have:

[5] 19	30	50	[2] 10
70	30	[7] 40	[2] 60
40	[8] 08	70	[10] 20

Minimum cost =

$$19\times5 + 2\times10 + 40\times7 + 60\times2 + 8\times8 + 20\times10$$

$$= 779 ₹$$

Step 2

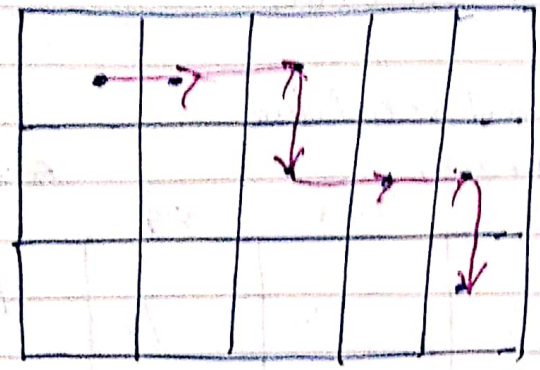
Check for degeneracy

IBFS has $m+n-1=6$, allocations in independent positions

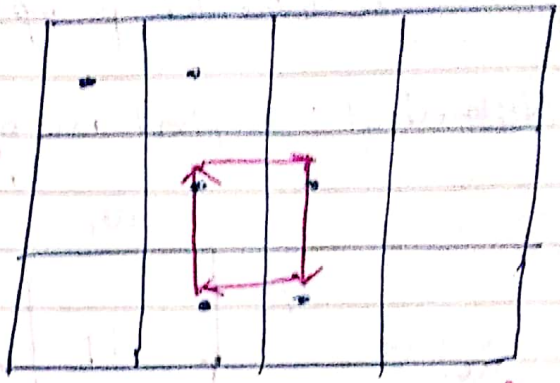
[m :- no. of Rows
n :- no. of Columns

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How to check Independent Position / Dependent position

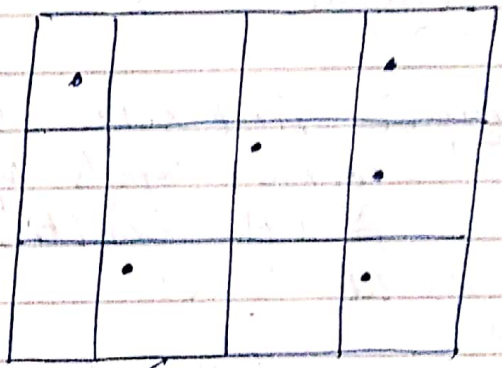


Don't make loop
↑ Independent Positions



Make the Loop
Dependent Position

5		2	
19	30	50	10
	7	2	
70	30	40	60
8		14	
40	08	70	20



✓ Independent Positions.

Hence optimality Test is satisfied.

Step 3: Calculation of u_i & v_j [occupied cells / where allocation is given]

(A) $u_i + v_j = c_{ij}$

19			10
		40	60
	8		20

$u_1 = 0$
$u_2 = 50$
$u_3 = 10$

$u_1 + v_1 = C_{11}$
$0 + v_1 = 19$
$\boxed{v_1 = 19}$

$u_1 + u_4 = C_{14}$
$0 + v_4 = 10$
$\boxed{v_4 = 10}$

u_3, u_4 (19) for rows u_2 (-2) v_3 (-10) v_4 (10)
u - for column

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$u_2 + v_4 = c_{24}$
$u_2 + 10 = 60$
$\boxed{u_2 = 50}$

$u_3 + v_4 = c_{34}$
$u_3 + 10 = 20$
$\boxed{u_3 = 10}$

$u_3 + v_2 = c_{32}$
$10 + v_2 = 08$
$\boxed{v_2 = -2}$

$u_2 + v_3 = c_{23}$
$50 + v_3 = 40$
$\boxed{v_3 = -10}$

Step 4: Calculation of cost differences for unoccupied cells.

(A) $d_{ij} = c_{ij} - (u_i + v_j)$

c_{ij}

·	30	50	·
70	30	·	·
40	·	70	·

$u_i + v_j$

				u_i
·	−2	−10	·	0
69	48	·	·	50
29	·	0	·	10
v_j → 19	−2	−10	10	

– where dots r located, no calculation.

Now d_{ij} =

·	32	60	·
1	−18	·	·
11	·	70	·

If more than one values are negative. Then pick most negative value.

Step 5 optimality Test $d_{ij} < 0$, i.e $d_{22} = -18$

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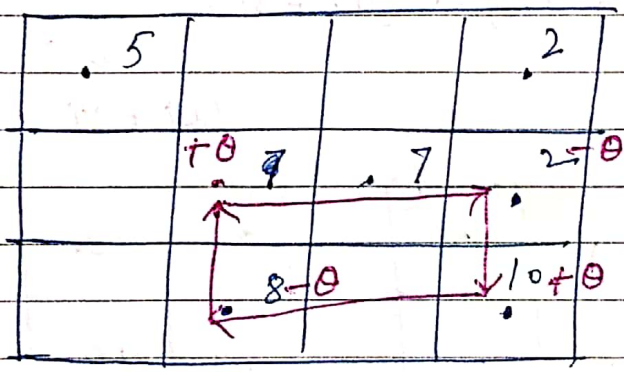
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Conditions:

→ When -ve value observed in cell, that means IBFS, value can further minimised so x_{22} is entering the basis.

→ But when +ve values observed in each cell that means IBFS is optimal.

Step 6: Construction of Loop and allocation of unknown quantity 'θ'.
- Loop will start from -ve value cell.



Direction will change from last allocation cell

minimum $[8-\theta, 2-\theta] = 0$

$8-\theta=0$ | $2-\theta=0$

$-\theta=-8$ | $-\theta=-2$

$\theta=8$ | $\theta=2$ ✓

mini value.

19(5)			10(2)
	30(2)	40(7)	—
	8(6)		20(12)

value coming Zero, so no allocation

(1) $m+n-1=6$ dependency
(2) No problem of

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mini. cost
$= 19\times5 + 10\times2 + 30\times2 + 40\times7 + 8\times6 + 20\times12$
$= 743$ ₹

(Again check optimality, either this cost is mini. or not)

Step 7. Improved Solution is $u_i + v_j = c_{ij}$

19			10	u_1
	30	40		$u_2 = 22$
	8		20	$u_3 = 0$

v_1 v_2 v_3 v_4
8 18 20

Allocation in each rows are same, so any value of u_i can put equal to zero.

$u_3 + v_2 = c_{32}$
$0 + v_2 = 8$
$v_2 = 8$

$u_3 + v_4 = c_{34}$
$0 + v_4 = 20$
$v_4 = 20$

$u_2 + v_2 = c_{22}$
$u_2 + 8 = 30$
$u_2 = 22$

$u_2 + v_3 = c_{23}$
$22 + v_3 = 40$
$v_3 = 18$

$u_1 + v_4 = c_{14}$
$u_1 + 20 = 10$
$u_1 = -10$

$u_1 + v_1 = c_{11}$
$-10 + v_1 = 19$
$v_1 = 29$

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Now $d_{ij} = c_{ij} - (u_i + v_j)$ [unoccupied cells]

c_{ij}

•	30	50	•
70	•	•	60
40	•	70	•

$u_i + v_j$ | $u_j \downarrow$

•	−2	8	•	−10
51	•	•	42	22
29	•	18	•	0
$v_j \rightarrow$ 29	8	18	20	

$d_{ij} = ?$

•	32	42	•
19	•	•	18
11	•	52	•

All values of $d_{ij} \geq 0$, and optimal solution is obtained (All values are positive of d_{ij}) with minimum cost i.e. 745!

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mini. cost

$= 19\times5 + 40\times2 + 30\times2 + 40\times7 + 8\times6 + 20\times12$

$= 743$ ₹

(Again check optimality, either this cost is mini. or not)

Step 7. Improved Solution is $u_i + v_j = c_{ij}$

19			10	u_1
	30	40		$u_2 = 22$
	8		20	$u_3 = 0$

v_1 v_2 v_3 v_4

8 18 20

Allocation in each rows are same, so any value of u_i can put equal to Zero.

$u_3 + v_2 = c_{32}$

$0 + v_2 = 8$

$\boxed{v_2 = 8}$

$u_3 + v_4 = c_{34}$

$0 + v_4 = 20$

$\boxed{v_4 = 20}$

$u_2 + v_2 = c_{22}$

$u_2 + 8 = 30$

$\boxed{u_2 = 22}$

$u_2 + v_3 = c_{23}$

$22 + v_3 = 40$

$\boxed{v_3 = 18}$

$u_1 + v_4 = c_{14}$

$u_1 + 20 = 10$

$u_1 = -10$

$u_1 + v_1 = c_{11}$

$-10 + v_1 = 19$

$\boxed{v_1 = 29}$

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Assignment Problem

workers / machines

Jobs	I	II	III	IV
A	2	3	5	3
B	10	7	13	14
C	3	2	1	10
D	3	5	4	6

Here 4 - Jobs
4 - machines /workers

Impl:
Assign one job to one worker only.
We need to minimise time to do the jobs
we need to minimise the cost to do the jobs

Sol

Hungarian method

Step I : 1) Select minimum value in each row/column then substract it from each element of corresp. ending row/column.

2) → In each column, there should be one zero.

Jobs \ workers	I	II	III	IV
A	0	1	3	1
B	3	0	6	7
C	2	1	0	9
D	0	2	1	3

Check for Zero in each row and column if found Zero is available in each row but one column is without Zero

i.e fourth Column.

0	1	3	0
3	0	6	6
2	1	0	8
0	2	1	2

Repeat Step One for fourth Column

Now in each row / column, there are Zeros.

Step 2 : Examine those rows which have exactly single Zero, make assignment (□) around it and cross all other Zeros in same Column.

~~0~~	1	3	0 ✓
3	[0]	6	6 ✓
2	1	[0]	8 ✓
[0]	2	1	2 ✓

Now check in each row, for unmarked Zero, if available then give assignment.

	I	II	III	IV
A	~~0~~	1	3	[0] ✓
B	3	[0]	6	6
C	2	1	[0]	8
D	[0]	2	1	2

Now in each row and column, one Zero is assigned. Then Stop.

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So optimal assignment has found.

A → IV
B → II
C → III
D → I

minimum assignment cost / Time = 3 + 7 + 1 + 3
= 14 ₹ / Time.

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